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FACTORS INFLUENCING THE VORTEX EFFECT IN HIGH-INTENSITY CYCLOTRONS

C. Baumgarten*, Paul Scherrer Institut, 5232 Villigen PSI, Switzerland

Abstract

We discuss factors that have potential influence on the space charge induced vortex motion of particles within high intensity bunches (curling of bunches, Gordon 1969) in isochronous cyclotrons. The influence of the phase slip due to deviations from strict isochronism determines if the bunches of a specific turn are above, below or at "transition", and hence whether stable vortex motion of the bunches is possible at all. Secondly there are possible longitudinal and transverse effects of rf acceleration, the former depending on the bunch phase ("bunching" or "debunching"), the latter depending on the gradient of the accelerating voltage. High accelerating voltages in the first turns call the applicability of adiabatic approximations and analytic methods into question. The influence of the rf acceleration is expected to be significant only at low beam energy, i.e. should have small or even negligible effect beyond the central region of compact machines.

INTRODUCTION

Due to their operation principle, isochronous cyclotrons provide no longitudinal focusing of particles within a bunch. In case of high beam intensity, the natural expectation would be that the presence of the space charge force has a defocusing effect in all directions. However, as explained by Gordon and others [1–8], the space charge force combines with the cyclotron specific coupling between longitudinal and transverse motion thus leading to a parasitic effective longitudinal focusing. This effective focusing was confirmed by bunch shape measurements in the PSI Injector II cyclotron [6,9,10] and allows to operate this machine at high intensities without flattop resonators. The phase of the former flattop resonators has been reversed in order to increase the energy gain per turn.

Because of the inherent nonlinearity and complexity of the problem, a full analytical treatment has not been found to date. Here we refer to a linear approximation that has been suggested [7,8] and used to understand the phenomenon in general. In Ref. [8] the linear model is used to develop a numerical code that allows to determine conditions for beam matching. This simple linear model effectively approximates the cyclotron by a constant focusing channel (CFC), and allows to derive some conclusions concerning the stability of the vortex effect. These conclusions have been compared with numerical studies using the particle-in-cell (PIC) code OPAL [11, 12]. The numerical simulations revealed that Gaussian beams which fulfill the linear matching conditions, are indeed (meta-) stable, but only under appropriate boundary conditions [8, 13]. Significant deviations from isochronism (strong phase excursions), for instance, are able to drive the beam out of the region of stability and may cause strong halo formation [13]. Here we present some results concerning effects of the acceleration voltage. These effects are expected to be important at low beam energy, i.e. during beam formation in the central region of the cyclotron.

CENTRAL REGION

A typical simplifying assumption used in previous studies is that of adiabatic acceleration, i.e. that the energy gain per turn (or per Dee gap) is small compared to the considered beam energy. This assumption is known to be questionable

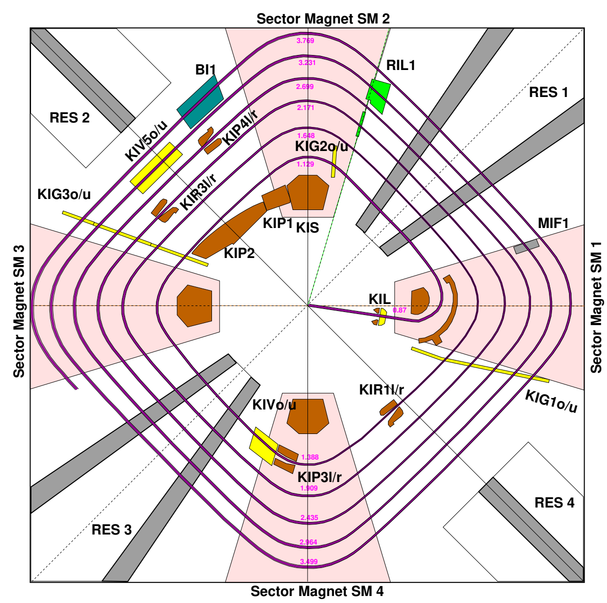

Figure 1: Layout of the central region of Injector 2.

in the first turns of Injector 2 (see Fig. 1): the DC beam of a Cockcroft-Walton preaccelerator passes a buncher before it is guided by an axial injection line towards the median plane. The injected beam energy is 870 keV, but the energy gain in the first resonator is of order ≈ 350 keV, and hence too large to safely presume adiabaticity.

We performed OPAL simulations of the acceleration of 2 mA beam for 3 different acceleration voltages with an initial distribution matched to a closed 1 MeV orbit in Injector 2. A too high accelerating voltage severely disturbs the vortex effect and leads to deformed bunches with increased halo formation as shown in Fig. 2. Figure 3 shows the number or particles versus distance from the bunch center and the corresponding integral. These results provide evidence that even in the case of well-matched beam injection, non-adiabatic acceleration causes halo formation and thus requires appro-

* christian.baumgarten@psi.ch

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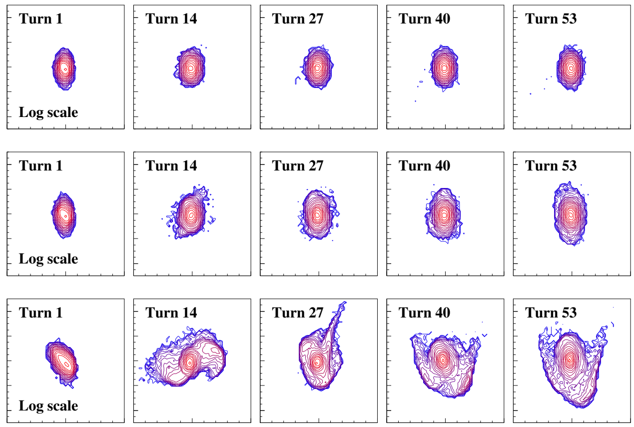

Figure 2: Top view contour lines (with logarithmic scale) of bunches with increasing turn number from left to right and (top to bottom) 25 %, 50 % and 100 % of nominal accelerating voltage, computed with OPAL.

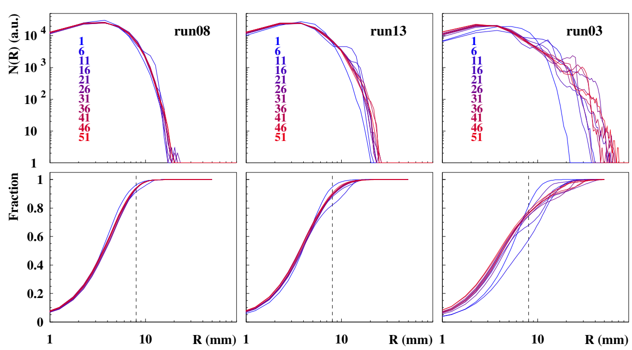

Figure 3: Radial distribution (top) and integral (bottom) for bunches with increasing turn number (blue to red). From left to right 25 %, 50 % and 100 % of nominal accelerating voltage, computed with OPAL. The dashed line is a guide to the eye.

priate beam collimation in order to achieve well separated turns and minimal extraction losses.

LINEAR MODEL

In order to discuss further disturbances due to acceleration and rf parameters, it is of advantage to review and extend the linear model developed in Ref. [8]. The effect of deviations from isochronism have been discussed in some detail in Ref. [8, 13] and are not considered here.

Since there are no linear terms that couple the longitudinal or transverse-horizontal motion to axial motion, the 6D problem can be split into a 2D treatment of axial and a 4D treatment of median plane motion. Let $\psi = (x, x', z, \delta)^T$ be the local median plane coordinates of a test particle relative to the reference orbit, then the linear terms of the CFC approximation $\psi' = \mathbf{F}\,\psi$ are given by the Hamiltonian matrix **F**:

$$\mathbf{F} = \begin{pmatrix} \cdot & 1 & \cdot & \cdot \\ -k_x + K_x & \cdot & K_{grad} & h \\ -h & \cdot & \cdot & \frac{1}{\gamma^2} \\ K_{grad} & \cdot & K_z\,\gamma^2 + K_{rf} & \cdot \end{pmatrix} \tag{1}$$

where $h = 1/r$ is the inverse radius, $k_x \approx h^2\,\gamma^2$ is the horizontal focusing term and $\gamma = (1 - v^2/c^2)^{-1/2}$ is the relativistic factor. K_x and K_z are the transverse horizontal and longitudinal space charge terms, respectively, K_{rf} represents linear (de-) bunching and K_{grad} represents the linear terms of the radial rf voltage gradient [14]. The space charge terms are, assuming optimal isochronism [8, 13, 15]:

$$\begin{aligned} K_x &= \frac{K_3\,(1-f)}{(\sigma_x+\sigma_y)\,\sigma_x\,\sigma_z} \\ K_z &= \frac{K_3\,f}{\sigma_x\,\sigma_y\,\sigma_z} \\ K_3 &= \frac{3\,q\,I\,\lambda}{20\,\sqrt{5}\,\pi\,\varepsilon_0\,c\,E\,\gamma\,(\gamma+1)} \end{aligned} \tag{2}$$

where I is the beam current, $\lambda = c/\omega_{rf}$, E is the kinetic energy and f is the “ellipsoidal form factor” which we assume to be $f \approx 1/3$. The eigenvalues $\pm i\,\Omega$ and $\pm i\,\omega$ of **F** are (see Fig. 4) [16]:

$$\begin{aligned} \Omega &= \sqrt{a/2}\,\sqrt{1 + \sqrt{1 - \chi^2}} \\ \omega &= \sqrt{a/2}\,\sqrt{1 - \sqrt{1 - \chi^2}} \end{aligned} \tag{3}$$

where $\chi^2 = 4\,b/a^2$ with

$$\begin{aligned} a &\equiv -\mathrm{Tr}(\mathbf{F}^2)/2 = \Omega^2 + \omega^2 \\ b &\equiv \mathrm{Tr}(\mathbf{F}^2)^2/8 - \mathrm{Tr}(\mathbf{F}^4)/4 = \Omega^2\,\omega^2 \end{aligned} \tag{4}$$

In order to have stable motion, the eigenfrequencies must be real which yields the condition $\chi^2 > 0$, hence $b > 0$, and finally (assuming $\gamma \approx 1$ in the central region)

$$b\,\gamma^2 = K_x\,(K_{rf} + K_z) - K_{grad}^2 > 0 \tag{5}$$

For the terms K_{rf} and K_{grad} we obtain:

$$\begin{aligned} K_{rf} &= h^2\,\frac{\gamma\,N_h\,q\,V\,\sin\phi}{2\,\pi\,(\gamma+1)\,E} \\ K_{grad} &= h\,\frac{\gamma\,q\,V'(r)\,\cos\phi}{2\,\pi\,(\gamma+1)\,E} \end{aligned} \tag{6}$$

where N_h is the harmonic number of the rf voltage, $q\,V(r)$ the maximal energy gain and ϕ the phase between bunch and rf. $V'(r) = \frac{dV}{dr}$ is the radial voltage gradient.

OPERATION WITH (DE-) BUNCHING PHASE

The K_{rf} term represents possible (de-) bunching effects, if the machine is not operated at zero phase. If the bunch lags behind $\phi > 0$, then particles ahead of the bunch center have a higher energy gain than particles in the bunch tail. Therefore tracking at starting phases of $\phi = 90°$ ($\phi = -90°$) can be used to analyze the debunching (bunching) phase without actual acceleration.

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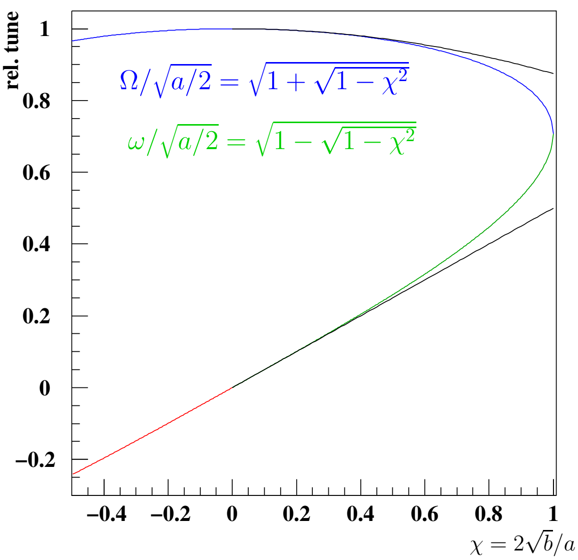

Figure 4: Horizontal and longitudinal tune in case of strong space charge.

For these cases, the linear model allows to compute the matching conditions for operation with a (de-) bunching phase. The results as shown in Fig. 5 indicate an increased (reduced) matched beam size for the debunching (bunching) phase. However, according to Eq. (5), the debunching phase

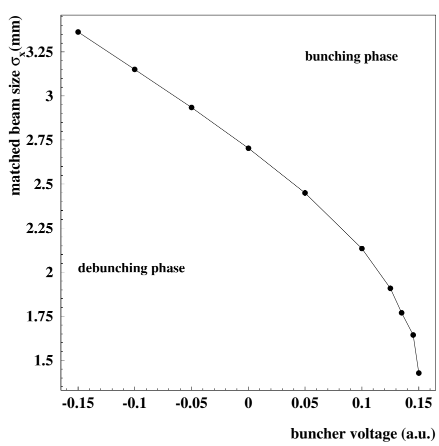

Figure 5: Matched horizontal bunch size σ_x versus buncher voltage for a 1 MeV beam coasting in Injector 2.

corresponds to $K_{rf} > 0$ and should theoretically provide a higher effective longitudinal tune and hence a better bunch stability. In order to test the expected effects with OPAL, we injected bunches at phases where no acceleration takes place, but (de-) bunching, that is, at $\phi = \pm 90°$. The model is confirmed by the plots in Figs. 6 and 7, which show that the bunching phase produces more halo, while the core stays more compact than in case of the debunching phase. Apparently the best compromize is to operate at $\phi \approx 0$ in order to avoid both, bunching and debunching effects.

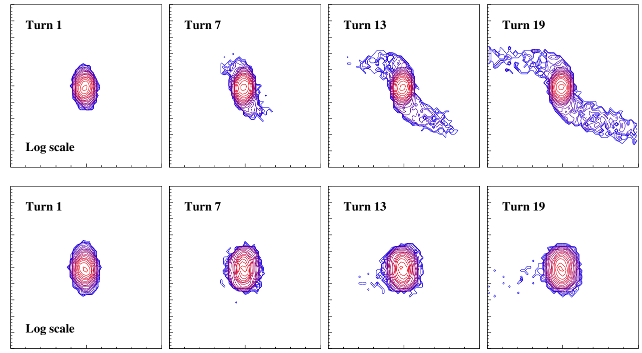

Figure 6: Top view contour lines (with logarithmic scale) of bunches with increasing turn number from left to right. Top: Bunching rf phase. Bottom: Debunching rf phase.

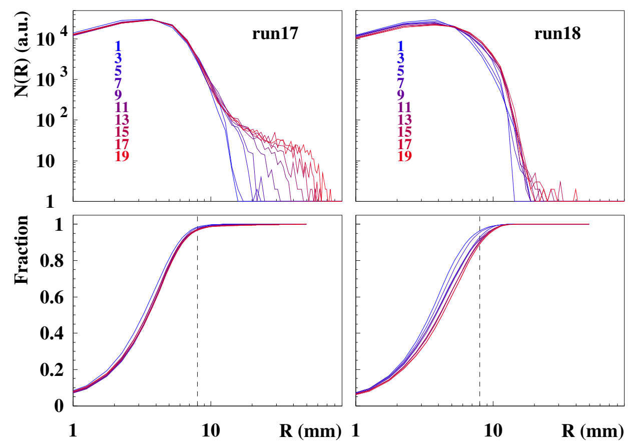

Figure 7: Radial distributions for bunching phase (left) and debunching phase (right).

STRONG VOLTAGE GRADIENTS

In order to investigate the influence of strong radial voltage gradients on the bunch, we generated a voltage profile with zero net voltage but significant voltage gradient at the bunch position. We then injected a matched bunch with (de-) accelerating phase $\phi = 0°$ ($\phi = 180°$), so that the bunch center "sees" no net voltage, but a positive (negative) voltage gradient. Figure 8 shows the resulting bunch form after some turns and Fig. 9 the corresponding radial distributions. The plots confirm that the presence of a voltage gradient reduces the bunch stability, especially in case of positive gradient. These results show that an attempt to circumvent the problem of adiabadicity by an rf design with a low starting voltage and high positive voltage gradient might result in new problems.

SUMMARY

We analyzed the possible effects of rf-acceleration on the vortex effect at low energy corresponding to the central region of a cyclotron with low injection energy as in case of the PSI Injector 2. OPAL simulations confirmed firstly that the matching condition sensitively depends on the assump-

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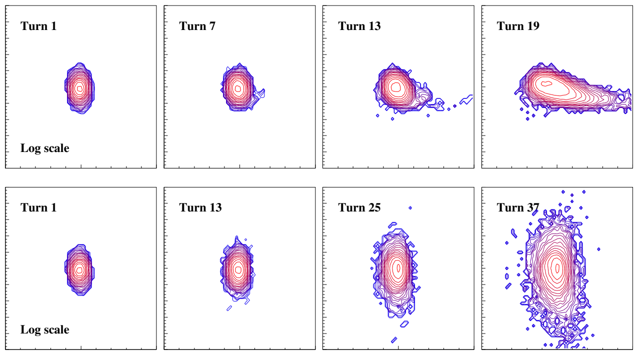

Figure 8: Radial distributions for positive (left) and negative (right) voltage gradient.

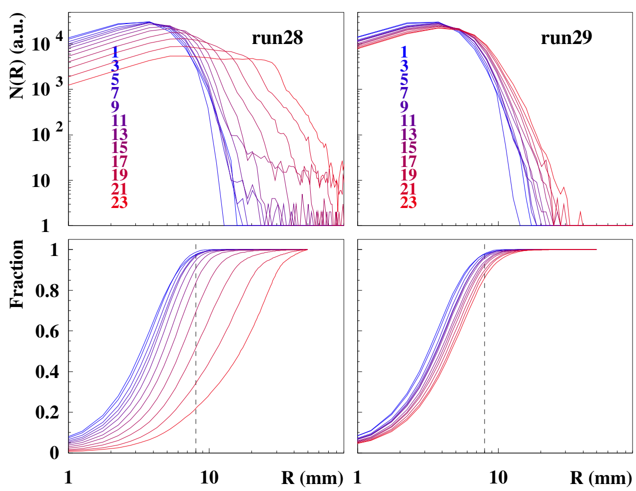

Figure 9: Radial distributions for positive (left) and negative (right) voltage gradient.

tion of adiabaticity. Secondly we found that (de-) bunching effects do not support the use of the vortex effect, i.e., do not significantly improve bunch stability. Finally we found that also strong voltage gradients have a potentially destructive effect on the vortex effect.

The latter effects can be avoided by an appropriate design of the rf system and/or by operation at the optimal phase $\phi = 0°$. The destructive effects of weak (de-) bunching or weak voltage gradients, respectively, become effective only after several turns. If the accelerating voltage is large, the beam will quickly achieve enough energy for these effects to be negligible.

To provide adiabaticity however, requires slow acceleration, i.e., a *small* accelerating voltage and is therefore in direct contradiction to the requirement of a low turn number and high turn separation. We conclude that – in case of electrostatic extraction – it is unavoidable to use an appropriate beam collimation system in the central region in support of proper beam formation and for the removal of beam halo. This of course requires sufficient energy gain and turn distance in order to place the collimators within the central region.

We believe that our results confirm the need for comprehensive numerical studies with suitable codes like OPAL in all cases where the use of the vortex effect is essential in order to achieve the design specifications of a high intensity cyclotron.

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